

# Multibin long-range correlations

A.Bialas and K.Zalewski

H.Niewodniczanski Institute of Nuclear Physics

Polish Academy of Sciences\*

and

M.Smoluchowski Institute of Physics

Jagellonian University†

January 11, 2011

PACS: 25.75.Dw, 25.75.Gz

keywords: long-range correlations, particle production

## Abstract

A new method to study the long-range correlations in multiparticle production is developed. It is proposed to measure the joint factorial moments or cumulants of multiplicity distribution in *several* (more than two) bins. It is shown that this step dramatically increases the discriminative power of data.

## 1 Introduction

It is now widely recognized that long-range correlations (LRC) in rapidity give information about the early stages of the collision. Indeed, such correlations cannot appear at late stages in the evolution of the produced system when longitudinal expansion separated the particles by large distances. Just after the collision, however, the system is small enough for the correlations to extend through the whole system.

---

\*Address: Radzikowskiego 152, 31-342 Krakow, Poland

†Address: Reymonta 4, 30-059 Krakow, Poland; e-mail:bialas@th.if.uj.edu.pl;

A special case of LRC are forward-backward correlations where one compares particle distributions in two intervals located symmetrically in the forward and backward hemispheres. They were extensively studied since early times of high-energy physics [1].

One of the interesting issues in particle production is the question if the produced particles "remember" the colliding projectiles, their energies, momenta and quantum numbers. Obviously the answer depends on the kinematic region we are considering. Close to the fragmentation region, the influence of the projectile on the produced particle spectrum is naturally expected. In the central rapidity region, far from the projectile fragmentation, the question remains open. On the theoretical side there is no consensus and various models give different answers.

An excellent review of models can be found in [1] (see also, [2]), therefore here we only quote some examples. With respect to the question of the number and structure of particle sources, they may be divided into three categories. In the first one, originating from the famous Landau and Feynman papers [3, 4], particles produced in the central rapidity region are decoupled from the projectiles. Thus the source of particles is symmetric with respect to  $y = 0$ . In the second class, like the wounded nucleon model [5], particles are produced by quasi-independent emission from the two colliding objects. In this case particles in the central region come from two sources, naturally asymmetric ones [6, 7]. There is of course also a third class which combines the two pictures, a typical example being the dual-parton model [8].

These various mechanisms can be tested (and verified) by studying the forward-backward correlations. The essential point is that correlations for one symmetric source are generally much stronger than those induced by two asymmetric ones [2, 9]. Following this general idea we recently proposed a systematic method of investigation of the forward-backward correlations in *symmetric* hadronic and heavy ion collisions [2]. It was shown that such investigations allow to verify how many independent sources of particles contribute to the observed distributions.

In the present paper we generalize these results in two respects:

- (i) we abandon the requirement of symmetry and consider the general case of asymmetric processes and thus also asymmetric sources;
- (ii) We suggest to measure and compare particle distributions in more than two intervals<sup>1</sup>.

---

<sup>1</sup>Measurements in three intervals were considered in [10] and [11].

This generalization of the problem allows to undertake a general discussion of LRC and thus extends its application to other processes, like, e.g. lepton-nucleon, hadron-nucleus and asymmetric nucleus-nucleus collisions.

We consider measurements of multiplicity moments in  $B$  intervals. Following our previous paper [2], we assume that particles are produced by independent sources and that the population of particles in  $B$  bins from one source is random, i.e. it is described by the multinomial distribution<sup>2</sup>.

We then evaluate the number of measurable (factorial) moments of the distribution and compare it with the number of parameters in the system. This allows to draw our main conclusion: the discriminating power of the method increases dramatically with increasing number of intervals in which the measurements are performed.

In the next section we present the mathematical structure of our approach. In Section 3 the number of possible measurements and number of independent parameters are evaluated for the general case of particle production from independent sources and a measurement in  $B$  intervals. An important special case, when the number of independent moments is reduced by symmetry, is discussed in the Appendix. In Section 4 the explicit formulae giving the factorial cumulants for two models with fixed numbers of sources are derived and commented upon. The general formulae and two examples of models with fluctuating numbers of sources are discussed in Section 5. The summary of the results can be found in the last section.

## 2 Formulation of the problem

Following the assumptions explained in Introduction, we write the generating function for the particle distributions in the  $B$  bins in the form

$$\Phi(z_1, \dots, z_B) = \left\langle \prod_{i=1}^N \phi_i^{w_i} (p_{1i}z_1 + \dots + p_{Bi}z_B) \right\rangle, \quad (1)$$

where  $\phi_i$  is the generating function for the  $i$ -th source,  $p_{ki}$  is the probability that the  $i$ -th source sends particles into the  $k$ -th bin and  $w_i$  is the number of sources of type  $i$ . The angular brackets denote averaging over the multiplicities  $w_i$  (they can be omitted if the  $w_i$  do not fluctuate). Since the generating

---

<sup>2</sup>This assumption which may be understood as the definition of a "source" is accepted in practically all published discussions of the forward-backward correlations, see e.g. [12]. For an extensive list of references, see [1].

functions  $\phi_i$  do not have to be all different, one can assume without loss of generality that each of the numbers  $w_i$  can only take the value zero or one (Section 5.1), but sometimes it is more convenient to assume that  $w_i$  can be any nonnegative integer (Section 5.2). The measurable (factorial) moments are given by

$$F_{i_1, \dots, i_B} \equiv \left\langle \prod_{j=1}^B \frac{n_j!}{(n_j - i_j)!} \right\rangle = \frac{\partial^r \Phi(z_1, \dots, z_B)}{\partial z_1^{i_1} \dots \partial z_B^{i_B}}. \quad (2)$$

where  $n_j$  is the number of particles in bin  $j$ . Here and henceforth all derivatives are taken at  $z = z_1 = \dots z_B = 1$ . Note that

$$\frac{d^r \phi_n(z)}{dz^r} \equiv F_n^{(r)} \quad (3)$$

is the  $r$ -th factorial moment of the distribution of the total number of particles sent by source  $n$  to all bins.

If the numbers and nature of the sources do not fluctuate, it is advantageous to introduce the cumulants

$$f_{i_1 \dots i_B} = \frac{\partial^r \log \Phi(z_1, \dots, z_B)}{\partial z_1^{i_1} \dots \partial z_B^{i_B}} \quad (4)$$

which, as is easily derived from (1), can be expressed as

$$f_{i_1 \dots i_B} = \sum_{n=1}^N p_{1n}^{i_1} \dots p_{Bn}^{i_B} f_n^{(r)}, \quad (5)$$

where

$$f_n^{(r)} = \frac{d^r \log \phi_n(z)}{dz^r} \quad (6)$$

are cumulants of the distribution produced by the  $n$ -th source.

Let us also note here that using (5) and the identity

$$\sum \frac{r!}{j_1! \dots j_B!} p_1^{j_1} \dots p_B^{j_B} = (p_1 + \dots + p_B)^r = 1, \quad (7)$$

one finds the very useful relation

$$\sum \frac{r!}{j_1! \dots j_B!} f_{j_1, \dots, j_B} = \sum_{n=1}^N f_n^{(r)}. \quad (8)$$

### 3 Counting of parameters

Consider a general situation of  $N$  groups of independent sources, all sources in one group being identical, and  $B$  bins. No symmetry relations among groups are assumed.

We first evaluate the number of moments which can be measured. To this end we observe that each moment has  $B$  indices:  $F_{i_1 i_2 \dots i_B}$ . Define the rank  $r$  of the moment as

$$r = i_1 + i_2 + \dots + i_B. \quad (9)$$

The number of moments at given  $r$  and  $B$ ,  $m(r, B)$ , is the solution of the well-known combinatorial problem: in how many ways can one distribute  $r$  identical objects among  $B$  boxes:

$$m(r, B) = \frac{(r + B - 1)!}{r!(B - 1)!} \rightarrow \sum_{r=1}^{r_{max}} m(r, B) = \frac{(B + r_{max})!}{B!r_{max}!} - 1 \quad (10)$$

The next thing we want to know is the number of parameters in the model. First, there are  $N(B - 1)$  independent probabilities. In addition we need also, for each kind source, the derivatives of order up to  $r_{max}$  of the multiplicity generating function  $[\phi_n(z)]^{w_n}$ :

$$\tilde{F}_n^{(r)} = \frac{d^r}{dz^r} [\phi_n^{w_n}(z)]. \quad (11)$$

They are polynomials in the random variable  $w_n$ .

The expressions for the measurable moments of order  $r$  contain the averages

$$\langle \tilde{F}_1^{(r_1)} \dots \tilde{F}_N^{(r_N)} \rangle, \quad \sum_{n=1}^N r_n = r. \quad (12)$$

When the multiplicity distribution for sources is not known, each of these averages is an independent parameter. Using the same combinatorial formulas as before, we thus find that the number of independent parameters is

$$P(B, N, r_{max}) = N(B - 1) + \frac{(r_{max} + N)!}{r_{max}!N!} - 1. \quad (13)$$

When the distribution of numbers of sources  $W(w_1, \dots, w_N)$  is known, all averages (12) are determined in terms of  $F_n^{(r_n)}$  and therefore the number of independent parameters is

$$P(B, N, r_{max}) = N(B + r_{max} - 1). \quad (14)$$

Thus we finally obtain for the number of parameter-independent constraints between the measurable quantities

$$C(N; B; r_{max}) = \frac{(B + r_{max})!}{B!r_{max}!} - 1 - P(B, N, r_{max}) \quad (15)$$

where  $P(B, N, r_{max})$  is given by (13) or (14).

To obtain tests, we demand that  $C \geq 1$ . It is clear that for any  $N$  and  $r_{max} \geq 2$  one can always find  $B$  such that this condition is satisfied.

For practical reasons, one has to keep  $r_{max}$  rather small, say 2 or 3. In Table 1 we give the minimal number of bins necessary to obtain parameter-independent constraints.

Table 1

Minimal numbers of bins necessary to get predictions				
	Fixed number of sources		Fluctuating number of sources	
$N$	$r_{max} = 2$	$r_{max} = 3$	$r_{max} = 2$	$r_{max} = 3$
1	2	2	2	2
2	3	2	4	3
3	5	3	6	4

To illustrate possible applications of this general discussion, we present in the next two sections four examples of specific models of particle production which can be tested in this way.

## 4 Fixed number of sources

For a fixed number of sources the measurable cumulants are given by (5). Below we give two specific examples.

## 4.1 Landau model: one source

In the Landau model there is just one source of particles, resulting from hydrodynamic expansion of the remnant of the two projectiles (just after collision the remnant is concentrated at  $y_{cm} = 0$ ). For one source ( $N = 1$ ) already at  $r_{max} = 1$  the number of measurable moments is equal to the number of parameters. Therefore, it is possible to determine all the probabilities  $p_j$  from the moments (cumulants) of rank one. For each  $r > 1$  there is one more parameter,  $f^{(r)} = d^r[\log \phi(z)]/dz^r$ . Using (5), this parameter can be evaluated from any measured moment of rank  $r$ . Indeed, for one source we simply have

$$f_{j_1, \dots, j_B}^{(r)} = p_1^{j_1} \dots p_B^{j_B} f^{(r)} \quad (16)$$

where the subscript denoting the source was dropped. Since all probabilities are already determined from the moments of rank one, this formula allows to evaluate  $f^{(r)}$  and thus all other measurable cumulants of rank  $r$ .

## 4.2 Deep inelastic scattering: two sources

In deep inelastic scattering there are at least two different sources: the proton and photon remnants. It is thus interesting to investigate if these two sources are sufficient to describe the data. In this section we show that the hypothesis of two sources gives indeed strong constraints on particle correlations.

Following the argument of Section 3, we consider  $B$  bins located anywhere along the direction of the incident photon. We thus have  $2(B + r_{max} - 1)$  parameters (the probabilities  $p_{j\gamma}$ ,  $p_{jP}$  and the cumulants  $f_\gamma^{(r)}$ ,  $f_P^{(r)}$ ). Let us denote by  $f_j^{(r)}$  the measurable cumulant of order  $r$  of the distribution of particles in the bin  $j$ . We show below how, using the measured cumulants  $f_j^{(r)}$  for  $r \leq 2$ , one can determine all the probabilities.

Since the sum rule (8) allows to determine the sum of the cumulants  $f_+^{(r)} = f_\gamma^{(r)} + f_P^{(r)}$  for any  $r$ , we are left with with  $r$  free parameters  $f_-^{(r)} = f_\gamma^{(r)} - f_P^{(r)}$  which should be sufficient to predict the correct values of the other measured cumulants. Instead of the parameters  $p_{j\gamma}$   $p_{jP}$  it is more convenient to use

$$p_{j\pm} = p_{j\gamma} \pm p_{jP}. \quad (17)$$

As already mentioned, for each  $r$  the parameter  $f_+^{(r)}$  can be obtained

directly from formula (8). For  $r = 1$  and  $r = 2$  we have

$$f_+^{(1)} = \sum_{i=1}^B \langle n_i \rangle ; \quad f_+^{(2)} = \sum_{i=1}^B \langle n_i(n_i - 1) \rangle + 2 \sum_{i>j}^B \langle n_i n_j \rangle \quad (18)$$

where  $n_i$  is the number of particles observed in bin  $i$ .

Let us consider first the cumulants of order one (they coincide with the moments of order one, i.e. average multiplicities). From formula (5) one gets

$$f_j^{(1)} = \frac{1}{2} (p_{j+} f_+^{(1)} + p_{j-} f_-^{(1)}) \rightarrow p_{j+} = \frac{2f_j^{(1)} - p_{j-} f_-^{(1)}}{f_+^{(1)}}, \quad (19)$$

which together with the sum rule for  $f_+^{(1)}$  eliminates  $B$  parameters.

Let us consider now the cumulants  $f_j^{(2)}$ . From (5) we have

$$4f_j^{(2)} = p_{j-}^2 f_+^{(2)} + 2p_{j-} p_{j+} f_-^{(2)} + p_{j+}^2 f_+^{(2)} \quad (20)$$

Using (19) to eliminate  $p_{j+}$  we get a quadratic equation for  $p_{j-}$ . The two solutions of this equation depend on the parameters  $f_-^{(1)}$  and  $f_-^{(2)}$ . Thus we get  $2^B$  possible sets  $p_{1-} \dots p_{B-}$ . Hopefully most of them can be eliminated by the obvious requirement that each  $p_{j-}$  must be real and that the following constraints must be satisfied.

$$|p_{j-}| < p_{j+}, \quad |p_{j-}| < 1, \quad \sum_{j=1}^B p_{j-} = 0. \quad (21)$$

Thus, if the model is consistent with data, i.e. solutions exist, all the probabilities are determined, though some ambiguities may be left.

For  $r_{max} = 2$  we have, in addition,  $\frac{1}{2}B(B-1)$  cumulants of the type  $f_{110\dots 0}$  which should be fitted with two parameters  $f_-^{(1)}$  and  $f_-^{(2)}$ . Increasing  $r_{max}$  by one, introduces two new parameters  $f_+^{(3)}$  and  $f_-^{(3)}$ . The former, however, is fixed by the sum rule (8) so that there are  $m(3, B)$  new cumulants, constrained by the sum rule which has already been used, to be fitted with one free parameter.

## 5 Fluctuating number of sources

When the number and nature of sources fluctuate, the discussion of LRC becomes rather involved. The reason is that the formulae expressing the



measurable moments in terms of the parameters of the model become complicated, as can be seen later in this section. In most models, however, the sources are not entirely arbitrary and thus these relations can be simplified.

We start with the general formulae for arbitrary number and nature of sources and then discuss two examples, suggested respectively by the dual parton model and by the wounded constituent model.

## 5.1 General formulae

Let us consider the generating function (1) with each  $w_i$  equal zero or one. Then

$$w_i(w_i - 1) = 0. \quad (22)$$

This greatly simplifies the differentiations. In fact

$$\frac{d^r}{dz^r} \phi_i^{w_i}(z) = w_i \frac{d^r}{dz^r} \phi_i(z) = w_i F_i^{(r)}. \quad (23)$$

Below we give the formulae for the measurable factorial moments of rank 1, 2 and 3. They are written assuming that only the first (for  $r = 1$ ), the first two (for  $r = 2$ ), or the first three (for  $r = 3$ ) bin indices are non-vanishing. Analogous formulae are of course valid for any other selection of bins, pairs of bins and triplets of bins.

$$\begin{aligned} F_{10\dots} &= \sum_{i=1}^N \langle w_i \rangle p_{1i} F_i^{(1)}, \\ F_{110\dots} &= \sum_{i=1}^N \langle w_i \rangle p_{1i} p_{2i} F_i^{(2)} + \sum_{i \neq j}^N \langle w_i w_j \rangle p_{1i} p_{2j} F_i^{(1)} F_j^{(1)} \\ F_{1110\dots} &= \sum_{i=1}^N \langle w_i \rangle p_{1i} p_{2i} p_{3i} F_i^{(3)} + \sum_{i \neq j}^N \langle w_i w_j \rangle p_{1i} p_{2i} p_{3j} F_i^{(2)} F_j^{(1)} + \\ &\quad \sum_{i \neq j \neq k \neq i}^N \langle w_i w_j w_k \rangle p_{1i} p_{2j} p_{3k} F_i^{(1)} F_j^{(1)} F_k^{(1)}, \end{aligned} \quad (24)$$

where  $F_i^{(r)}$  is the  $r$ -th factorial moment of the distribution of particle from source  $i$  (c.f. (3)). When some indices coincide, it is enough to change

correspondingly the bin indices of the probabilities  $p$ . For instance,

$$F_{20\dots} = \sum_{i=1}^N \langle w_i \rangle p_{1i}^2 F_i^{(2)} + \sum_{i \neq j}^N \langle w_i w_j \rangle p_{1i} p_{1j} F_i^{(1)} F_j^{(1)}. \quad (25)$$

## 5.2 Dual parton model

For a general nucleus-nucleus collision we have a certain number  $N_L$  of identical sources moving left, a number  $N_R$  of identical sources moving right and  $N_C$  identical symmetric sources. These numbers fluctuate from event to event and their distribution depends also on the centrality of the collision. The left and right moving sources are mirror images of each other with respect to cm rapidity.

We consider the case where the bins are also selected to be symmetric with respect to  $y_{cm} = 0$ . Then if  $\phi_a(p_{1a}z_1 + \dots + p_{Ba}z_B)$ , where  $a$  stands for asymmetric, is the generating function for the multiplicity distributions in the bins  $1, \dots, B$  of the particles originating from a left moving source, then  $\phi_a(p_{Ba}z_1 + \dots + p_{1a}z_B)$  is the corresponding generating function for the particles originating from a right moving source.

Let us denote by  $w_L$ ,  $w_R$ ,  $w_C$  the numbers of left moving, right moving and central sources. In [2] we discussed mostly the case of two bins and fixed  $w_L = w_R$  and  $w_C$ . Here we assume an arbitrary number of bins and a general joint probability distribution  $W(w_L, w_R, w_C)$  which, however, can be evaluated, e.g. by the Glauber method (the result will, naturally, depend on the model adopted for particle production). Then the overall generating function for the multiplicity distributions in the  $B$  bins is

$$\begin{aligned} \Phi(z_1, \dots, z_B) = & \sum_{w_L, w_R, w_C} W(w_L, w_R, w_C) \\ & [\phi_a(p_{1a}z_1 + \dots + p_{Ba}z_B)]^{w_L} [\phi_a(p_{Ba}z_1 + \dots + p_{1a}z_B)]^{w_R} \\ & [\phi_C(p_{1C}z_1 + \dots + p_{BC}z_B)]^{w_C}. \end{aligned} \quad (26)$$

We will denote the probabilities by  $p_{iA}$  where  $A = L, R, C$ . Although the probabilities  $p_{iR}$  can be expressed by the probabilities  $p_{iL}$ , this redundancy in the notation makes the following formulae much shorter. Similarly, the derivatives (11) are denoted by  $\tilde{F}_A^{(r)}$ . Using this notation, the explicit expressions for the measurable factorial moments, obtained by differentiation of (26), read

$$\begin{aligned}
F_i^{(1)} &= \sum_A p_{iA} \langle \tilde{F}_A^{(1)} \rangle; \\
F_{ij}^{(2)} &= \sum_A p_{iA} p_{jA} \langle \tilde{F}_A^{(2)} \rangle + \sum_{A \neq B} p_{iA} p_{jB} \langle \tilde{F}_A^{(1)} \tilde{F}_B^{(1)} \rangle; \\
F_{ijk}^{(3)} &= \sum_A p_{iA} p_{jA} p_{kA} \langle \tilde{F}_A^{(3)} \rangle + \\
&+ 3 \sum_{A \neq B} [p_{iA} p_{jB} p_{kB} + p_{jA} p_{kB} p_{iB} + p_{kA} p_{iB} p_{jB}] \langle \tilde{F}_A^{(1)} \tilde{F}_B^{(2)} \rangle + \\
&+ \sum_{A \neq B \neq C \neq A} p_{iA} p_{jB} p_{kC} \langle \tilde{F}_A^{(1)} \tilde{F}_B^{(1)} \tilde{F}_C^{(1)} \rangle. \tag{27}
\end{aligned}$$

The parameters of the model are the  $2(B-1)$  probabilities and the averages (12). Their number is given by (13) or (14) with  $N = 2$ .

Since formulae (27) are rather complicated, it seems that in absence of other constraints, the best way to proceed is to try to fit them by minimizing the  $\chi^2$ . If the fit works, the resulting values of the probabilities and of the factorial moments give information about the properties of the sources.

For  $pp$  scattering  $w_L = w_R = 1$  and thus the relations are much simpler. As they may be easily obtained from (27), we discuss here only the number of parameter-independent constraints. The number of possible measurements is given in the Appendix. The number of parameters is  $B + B/2 - 2 + 2r_{max}$  for  $B$  even and  $B + (B+1)/2 - 2 + 2r_{max}$  for  $B$  odd. One can see that for  $r_{max} = 3$  there are already 3 constraints for  $B = 3$  and 8 constraints for  $B = 4$ . If one wants to restrict the measurements to  $r_{max} = 2$ , it is necessary to measure distributions in at least 5 bins. Then one obtains 2 constraints.

### 5.3 Wounded constituent model

In the wounded constituent model, particles are emitted by the wounded constituents moving left or right, thus there are no central sources. The relevant formulae can be obtained from (27) by putting  $\tilde{F}_C^{(r)} = 0$ .

They can be written in the form:

$$\begin{aligned}
F_i^{(1)} &= p_{iL} \langle \tilde{F}_L^{(1)} \rangle + p_{iR} \langle \tilde{F}_R^{(1)} \rangle; \\
F_{ij}^{(2)} &= p_{iL} p_{jL} \langle \tilde{F}_L^{(2)} \rangle + p_{iR} p_{jR} \langle \tilde{F}_R^{(2)} \rangle + (p_{iL} p_{jR} + p_{iR} p_{jL}) \langle \tilde{F}_L^{(1)} \tilde{F}_R^{(1)} \rangle; \\
F_{ijk}^{(3)} &= \{ p_{iL} p_{jL} p_{kL} \langle \tilde{F}_L^{(3)} \rangle + (L \rightarrow R) \} +
\end{aligned}$$

$$+ \left\{ 3[p_{iL}p_{jR}p_{kR} + p_{jL}p_{kR}p_{iR} + p_{kL}p_{iR}p_{jR}] \langle \tilde{F}_L^{(1)} \tilde{F}_R^{(2)} \rangle + (L \leftrightarrow R) \right\}. \quad (28)$$

The consequences of these formulae are different for symmetric (e.g.  $Au - Au$  collision) and asymmetric (e.g.  $d - Au$  collision) processes.

When the probability distribution  $W(w_L, w_R)$  is known, the number of parameters at a given  $r_{max}$  is  $B + r_{max} - 1$ . For asymmetric processes the number of possible measurements is given by (10) and for symmetric processes the relevant formulae are given in the Appendix. Already for  $B = 3$  and  $r_{max} = 2$  one obtains 2 parameter-independent constraints for symmetric and 5 for asymmetric processes.

When  $W(w_L, w_R)$  is not known, the various averages of the moments  $\langle \tilde{F}_L^{(s)} \tilde{F}_R^{(r-s)} \rangle$ , ( $s = 0, \dots, r$ ), have to be fitted from data at every  $r \leq r_{max}$ . At given  $r$  the number of independent averages is  $r + 1$  for the asymmetric case and for the symmetric case it is  $r/2 + 1$  for  $r$  even and  $(r + 1)/2$  for  $r$  odd. It is remarkable that already at  $B = 3$  parameter-independent constraints exist. For symmetric processes one obtains 1 constraint for  $r_{max} = 2$  and 5 constraints for  $r_{max} = 3$ . For asymmetric processes the corresponding numbers are 2 and 8.

## 6 Summary

Extending the ideas formulated in [2] (see also [9]), a new method to study the long-range correlations in particle production is developed. The new proposition is to measure the factorial moments and/or cumulants in *several* bins, as opposed to previous studies which were mostly restricted to just two bins (see, however, footnote 1). It was shown that increasing the number of bins magnifies dramatically the possibility of discriminating between various models of particle production.

The discriminative power of the method was analyzed in the most general way. Apart from this general treatment, four specific (and popular) models of particle production were discussed. It was shown that the suggested measurements provide strong constraints on all of them.

The method seems rather general and flexible. It can be applied to symmetric, as well as to asymmetric processes. It can be used to study distributions in various kinematic variables (e.g. rapidity and transverse momentum [2]). Finally, it does not require full acceptance of the detector.

We conclude that studies of long-range correlations in multiparticle production may become a powerful instrument in investigations of particle production mechanisms at high energy.

## 7 Appendix

In this appendix we calculate the number of moments of order  $r$  for the reflection symmetric case, i.e. when

$$F_{i_1 \dots i_B} \equiv F_{i_B \dots i_1}. \quad (29)$$

We will denote the number of these moments by  $m_S(r, B)$ . Let us call symmetric the moments for which the ordered sets  $\{i_1 \dots i_B\}$  and  $\{i_B \dots i_1\}$  coincide. The number of such moments will be denoted  $S(r, B)$ . The constraint (29) does not affect the number of symmetric moments, but reduces the number of the other independent moments by a factor of two. Thus

$$m_S(r, B) = \frac{1}{2}[m(r, B) + S(r, B)], \quad (30)$$

where  $m(r, B)$  is given by formula (10) and  $S(r, B)$  remains to be calculated.

Let us begin by the case when  $B = 2K + 1$ , where  $K$  is an integer. Then the generic form of a symmetric moment is  $F_{i_1, \dots, i_K, n, i_K, \dots, i_1}$ . Therefore, the number of such moments can be calculated as follows. Include all the nonnegative integers  $p$  such that  $n = r - 2p \geq 0$ . Notice that for each  $p$  there are  $m(p, K)$  moments, thus in this case

$$S(r, B) = \sum_p \binom{p + K - 1}{p}. \quad (31)$$

The case  $B = 2K$  reduces to the previous one with the constraint that  $n = 0$ . For  $r = 2I + 1$ , where  $I$  is an integer, there are no solutions for  $p$ , therefore

$$S(r, B) = 0, \quad (32)$$

while for  $r = 2I$  the only solution is  $p = I$ , so that

$$S(r, B) = \binom{I + K - 1}{I}. \quad (33)$$

## 8 Acknowledgements

This investigation was supported in part by the grant N N202 125437 of the Polish Ministry of Science and Higher Education (2009-2012).

## References

- [1] For a review, see W.Kittel and E.A.De Wolf, *Soft Multihadron Dynamics*, World Scientific (2005).
- [2] A.Bialas and K.Zalewski, *Phys. Rev. C* 82 (2010) 034911.
- [3] L.Landau, *Izv. Akad. Nauk Ser. Fiz.* 17 (1953) 51.
- [4] R.P. Feynman, *Phys. Rev. Lett.* 23 (1969) 1415.
- [5] A.Bialas, M.Bleszynski and W.Czyz, *Nucl.Phys.* B111 (1976) 461; A.Bialas, W.Czyz and W.Furmanski, *Acta Phys. Pol.* B8 (1977) 585.
- [6] A.Bialas and W. Czyz, *Acta Phys. Pol.* B36 (2005) 905.
- [7] A. Bialas and A.Bzdak, *Acta Phys. Pol.* B38 (2007) 159; *Phys. Lett.* B649 (2007) 263; *Phys. Rev. C* 77 (2008) 034908. For a review, see A.Bialas, *J.Phys.* G35 (2008) 044053.
- [8] For a review, see A.Capella, U.Sukhatme, C.-I.Tan and J.Tran Thanh Van, *Phys. Rep.* 236 (1994) 225.
- [9] A. Bzdak, *Acta Phys. Pol.* B41 (2010) 151; *Acta Phys. Pol.* B41 (2010) 2471; *Acta Phys. Pol.* B40 (2009) 2029.
- [10] B.I.Abelev et al., STAR collaboration, *Phys. Rev. Letters* 103 (2009) 172301; T.J.Tarnowsky, arXiv: 0807.1941.
- [11] T.Lappi and L. McLerran, *Nucl. Phys.* A832 (2010) 330.
- [12] T.T.Chou and C.N.Yang, *Phys. Lett.* B135 (1984) 175; P.Carruthers and C.C.Shih, *Phys. Lett.* B165 (1985) 209; W.A.Zajc, *Phys. Lett.* B175 (1986) 219; J.Benecke, A.Bialas, and S.Pokorski, *Nucl.Phys.* B135 (1976) 488; Erratum: *ibid.* B115 (1976) 547.